University of California, Berkeley
Physics 110A, Section 2, Spring 2003 (*Strovink*)

PROBLEM SET 3

1.
Three identical parallel conducting plates have a length and width that are large compared to their thickness or mutual separation. Plate 1 is on the bottom and plate 3 is on the top. The plates carry total charge Q_1, Q_2, and Q_3, respectively. On each plate n, the charge is divided between the top-surface charge Q_n^t and the bottom surface charge Q_n^b. What are these six charges?

2.
(a.)
Imagine that the electron is a uniformly charged sphere of radius r_a. Equating its electrostatic self-energy with its mass energy mc^2, and using the known electron charge and mass, calculate this classical electron's radius r_0 (in units of fm = 10^{-15} m). [Actually – to much higher accuracy than r_0 – the electron, other electron-like objects such as muon and tau, and quarks appear to be point particles – even the top quark, which is $5\frac{1}{2}$ orders of magnitude heavier.]
(b.)
Instead assume that all of the classical electron's charge lies on the surface of a sphere. Again equating electrostatic to mass energy, and taking $V = 0$ at infinity, calculate the electrostatic potential (in megavolts) on this classical electron's surface.

3. A fully charged automobile battery can deliver 500 ampere-hours. Using inexpensive materials, you build a capacitor to store the same amount of charge. In alternating layers, you stack 1 m^2 sheets of kitchen foil and trash-bag polyethylene, each of 0.025 mm ("1 mil") thickness. You connect in parallel alternate aluminum sheets to form the two capacitor electrodes.

According to tables, the dielectric constant ϵ/ϵ_0 of polyethylene is equal to 2.3; a 0.025 mm thickness of polyethylene will stand off up to 500 volts. To avoid short circuits through imperfections, you decide to run the capacitor at 250 volts instead.

How high (in m) is your stack?

4. An infinite thin wire parallel to the x axis carries a line charge λ coul/m. It lies a distance D above an infinite grounded conducting plane $z = 0$. Calculate the force per unit x by which the plane is attracted to the wire.

5.+6.
(a.)
Use the method of images to evaluate

$$\mathcal{E} \equiv |\vec{E}(0, 0, u)|$$

created when a static point charge q is located at $(0, 0, \epsilon)$ above an infinite grounded conducting plane $z = 0$. For $\epsilon \ll u$, use first-order Taylor series expansions to find the leading dependence of $\mathcal{E}$ upon u.
(b.)
Use the method of images to evaluate

$$\mathcal{E} \equiv \left|\vec{E}(0, u/\sqrt{2}, u/\sqrt{2})\right|$$

created when a static point charge q is located at $(0, \epsilon/\sqrt{2}, \epsilon/\sqrt{2})$ in the first quadrant between two infinite grounded conducting planes $z = 0$ and $y = 0$ that intersect on the x axis. For $\epsilon \ll u$, use second-order Taylor series expansions to find the leading dependence of $\mathcal{E}$ upon u.
(c.)
A pair of infinite grounded conducting planes intersects on the x axis with acute opening angle π/n, where n is an integer. Between the planes, a static point charge lies on the plane that bisects this small angle, at $x = 0$ and at a perpendicular distance ϵ from the x axis. Consider $\mathcal{E}$, defined as $|\vec{E}|$ at $x = 0$ and at a perpendicular distance u from the x axis on the same bisecting plane. For $\epsilon \ll u$, *guess* the leading dependence of $\mathcal{E}$ upon u.

7.+8.
(a.)
An infinitely long piece of grounded conducting "square tubing", centered on the y axis, has inside boundaries defined by the sides $x = \pm L/2$ and $z = \pm L/2$. On its midplane $y = 0$, an insulating membrane is stretched flat and smeared with charge to create the electrostatic potential

$$V(x, y = 0, z) = V_0 \cos \frac{\pi x}{L} \cos \frac{\pi z}{L} \ .$$

Calculate the electric field on the positive y axis. With what characteristic length does its magnitude decay exponentially in y?
(b.)
If this were a 2D instead of a 3D problem (*i.e.* if nothing varied with z), would the electric field decay slower or faster in y?
(c.)
Returning to the 3D problem, calculate the charge density $\sigma(y > 0, z)$ on the inside surface $x = L/2$ of the square tubing. How much total charge was smeared on the insulating membrane?